Random and nonrandom anisotropy-induced crossover in vector spin glasses

G. Kotliar

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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We study the effect of Dyalozinsky-Moriya (DM) and single-ion (SI) cubic anisotropy on the vector spin-glass critical behavior 6—ε dimensions. These anisotropies, which induce very different symmetry-breaking patterns of the $O(m) \times \cdots \times O(m)$ symmetry of the m-vector spin-glass critical point, are more relevant than the temperature. We calculate the crossover exponent for the DM interaction, $\phi_D(\text{DM})$, to two-loop order. $\phi_D(\text{DM})$ is larger than the single-ion anisotropy crossover exponent $\phi_D(SI)$. We find that $\phi_D(SI)$ is equal to $\gamma$, the nonlinear susceptibility exponent, to all orders in $\epsilon$.

I. INTRODUCTION

The effect of anisotropy on the spin-glass critical behavior has attracted much attention recently both theoretically and experimentally. Both uniaxial single-ion anisotropy and Dyalozinsky-Moriya anisotropy are relevant perturbations to the Heisenberg-Ising critical behavior. In finite magnetic fields, random anisotropy affects the shape of the spin-glass paramagnetic phase boundary. In the infinite-range model it crosses over from the Gabay-Toulouse behavior $T_E(H) - T_E(0) \sim H^2$ to the de Almeida-Thouless behavior $T_E(H) - T_E(0) \sim H^{2/3}$ via two intermediate scaling regimes $T_E(H) \sim T_E(0) \sim H^{1/3}$ and $T_E(H) \sim T_E(0) \sim \text{const}$ as the anisotropy is increased. Fisher and Sompolinsky showed that this behavior, which arises from a dangerously irrelevant variable, persists above eight dimensions and should disappear below dimension 6.

In metallic spin glasses like Cu$_{1-x}$Mn$_x$ the strongest anisotropy originates in the Dyalozinsky-Moriya (DM) interactions between the Mn magnetic moments. The strength of this interaction can be controlled by doping the alloy with a strong spin-orbit scatterer like gold. The effect of anisotropy on the critical temperature, the critical behavior, and the irreversibility lines has been studied experimentally using torque, linear, and nonlinear susceptibility measurements. While it is clear that anisotropy has a strong effect on the spin-glass critical behavior, the evidence for scaling is still inconclusive.

In this paper we study the effect (random) DM anisotropy and (nonrandom) single-ion (SI) uniaxial anisotropy on the spin-glass critical behavior using the $\epsilon$ expansion around six dimensions. We define and calculate the crossover exponents for both types of anisotropies. Both anisotropies turn out to be more relevant than the temperature which could make the Heisenberg critical behavior rather difficult to observe.

This paper is divided into five sections in Sec. II we derive the replicated Hamiltonian corresponding to the different anisotropies. In Sec. III, we set up the renormalization-group machinery needed to calculate the crossover exponents. The treatment in Sec. III follows Refs. 16–18 and is presented in detail to define the notation. The results of Sec. III are then applied to the DM anisotropy in Sec. IV and to the single-ion uniaxial anisotropy in Sec. V. We discuss the experimental relevance of these results in the conclusion.

II. THE MODELS AND THEIR SYMMETRIES

The vector spin-glass Hamiltonian is described as

$$H = \sum_{(i,j)} S_x^{i,j} S_y^{i,j} J_{xy}.$$  

$$S_x$$ is a vector spin and $J_{xy}$ scalar random variables with zero mean and variance $\langle J_{xy}^2 \rangle = J^2$. The effect of the DM interaction is studied adding a random interaction

$$H_{DM} = \sum_{(i,j)} D^{i,j}_{xy} S_x^i S_y^j.$$  

$x,y$ are site indices, $i,j$ are spin indices, and $D^{i,j}_{xy}$ an antisymmetric ($D^{i,j}_{xy} = -D^{j,i}_{xy}$) random matrix with zero average and variance $D^2 = \langle D^{i,j}_{xy} \rangle$. Equation (2) is a generalization of the DM interaction to m-vector spins.

For Heisenberg ($m = 3$) spins, this can be rewritten as $D^{i,j}_{xy} S_x^i S_y^j = K_{xy} (S_x^i S_y^j)^j$ with $K_{xy} = -\epsilon_{ijk} D^{i,j}_{xy}$ a random vector, which is the standard DM form.

Using the replica trick and the techniques of Harris et al., this model can be mapped onto an effective field theory with Lagrangian

$$L = L_0 + L_{DM},$$

$$L_0 = \frac{r_0}{4} Q^{i,j}_{\alpha \beta} Q^{i,j}_{\alpha \beta} + \frac{1}{4} \nabla Q^{i,j}_{\alpha \beta} \nabla Q^{i,j}_{\alpha \beta} - \omega Q^{i,j}_{\alpha \beta} Q^{i,j}_{\gamma \delta} Q^{\gamma \delta}_{\alpha \beta},$$

$$L_{DM} = \frac{r_1}{4} Q^{i,j}_{\alpha \beta} Q^{i,j}_{\alpha \beta} + \frac{r_2}{4} Q^{i,j}_{\alpha \beta} Q^{i,j}_{\alpha \beta}.$$

$\alpha, \beta = 1, \ldots, n$ is a replica index, $j = 1, \ldots, m$ is a spin index, and summation over repeated indices is implied. $Q^{i,j}_{\alpha \beta} = \langle S^{i,j}_{\alpha \beta} \rangle$ is the Edwards-Anderson order parameter and $\alpha, \beta$ denote identical replicas of the system. $L_0$ is invariant under replica-dependent rotations.
\[
Q_{ab}^{ij} \rightarrow R_{ab}^{ij} Q_{ab}^{(i)j} R_{ab}^{(j)i}, \quad R_{ab} \in O(m).
\]

This symmetry is the manifestation of the rotational invariance of the Hamiltonian, Eq. (1) in the replicated theory.\textsuperscript{19} \(L_{DM}\) is a symmetry breaking term which reduces the full \(O(m) \times \cdots \times O(m)\) symmetry of \(L\) to \(O(m)\). In fact, \(L_{DM}\) is the most general quadratic term consistent with the symmetry \(Q_{ab}^{ij} \rightarrow R_{ab}^{ij} Q_{ab}^{(i)j} R_{ab}^{(j)i}\), \(R \in O(m)\). This symmetry under replica-independent rotations reflects the isotropy of the Hamiltonian (1),(2) on the average.

This symmetry-breaking pattern,
\[
\{O(m) \times O(m) \times \cdots \times O(m)\}_n > O(m)
\]
(here \([\cdot ]_n\) means a direct product was taken \(n\) times) which is discussed in Secs. III and IV, should be contrasted with the one produced by a single-ion anisotropy term
\[
H_{SI} = \sum_x (S_x^z)^2
\]
\[
\{O(m) \times O(m) \times \cdots \times O(m)\}_n > [O(m-1) \times O(m-1) \times \cdots \times O(m-1)]_n.
\]

In the replicated theory, this term is mapped onto
\[
L_{SI} = \frac{r_1}{4} Q_{ab}^{(i)j} Q_{ab}^{(j)i} + \frac{r_2}{4} (Q_{ab}^{(i)j} Q_{ab}^{(j)i} + Q_{ab}^{(j)i} Q_{ab}^{(i)j}),
\]
which will be studied in Sec. V.

The Ising-Heisenberg crossover should be observable in the nonlinear susceptibility and the shape of the paramagnetic spin glass phase boundary. The nonlinear susceptibility
\[
\chi_{nl} = \chi(h,t) - \chi_0(t)
\]
behaves singularly as one approaches the phase boundary. Its scaling behavior has been studied by several authors\textsuperscript{10,20} and is summarized in Eq. (10).
\[
\chi_{nl}^{-1} = (t - t_{c} h^2)^{-1} f \left( \frac{h^2}{t_{c}^{\phi_2}}, \frac{d^2}{t_{c}^{\phi_3}}, \frac{d^3}{t_{c}^{\phi_3}} \right).
\]

\(t = (T - T_c)/T_c\) is the reduced temperature, \(h\) is a reduced magnetic field, and \(d\) is a dimensionless measure of the strength of the anisotropy. The exponents \(\phi_2, \phi_3\) will be extracted from the order parameter two-point vertex function in zero field,
\[
\lim_{h \to 0} h^\gamma \chi_{nl}^{-1} = \Gamma^{(2)} = \langle Q_{ab}^{(i)j} Q_{ab}^{(j)i} \rangle^{-1}.
\]

In a pure vector spin glass the transition line is given by
\[
t \sim h^2/\phi_1,
\]
with \(\phi_1 = \nu(d + 2 - \eta)/2\) the exponent of the Heisenberg fixed point.\textsuperscript{13} Assuming \(\phi_2 > \phi_3\), the random anisotropy then is felt when the anisotropy length crosses over with the magnetic length
\[
d^2 \sim (h^2)^{\phi_1/\phi_2}.
\]

The exponent \(\phi_2\) controls the singular part of the shift in the critical temperature as the anisotropy is varied,
\[
\frac{T_c(d) - T_c(0)}{T_c(0)} \sim (d^2)^{1/\phi_2}.
\]

### III. RENORMALIZATION OF THE ANISOTROPIC FIELD THEORY

To study the effect of the different anisotropies on the Heisenberg critical point we use the field theoretical renormalization group. The technique is standard and is pedagogically reviewed in Refs. 16–18.

The vector spin glass at its critical point is described by
\[
L_0 = \frac{1}{4} \nabla Q_{ij}^{a\beta} \nabla Q_{ij}^{a\beta} - \omega_B Q_{ij}^{a\beta} Q_{ij}^{a\alpha} Q_{kl}^{\alpha \beta}.
\]

deviations away from the critical point, and the DM interactions introduce insertions of the operators:
\[
A_1 = \frac{1}{4} Q_{ij}^{a\beta} Q_{ij}^{a\beta},
\]
\[
A_2 = \frac{1}{4} Q_{ij}^{a\beta} Q_{ij}^{a\beta},
\]
\[
A_3 = \frac{1}{4} Q_{ij}^{a\beta} Q_{ij}^{a\beta}.
\]

To treat the uniaxial single-ion anisotropy we will introduce operator insertions
\[
A_1 = \frac{1}{4} Q_{ij}^{a\beta} Q_{ij}^{a\beta},
\]
\[
A_2 = \frac{1}{4} \left( \langle Q_{ij}^{a\beta} \rangle^2 - \frac{1}{m - 1} \sum_{i \neq j} \langle Q_{ij}^{a\beta} \rangle^2 + \langle Q_{ij}^{a\beta} \rangle^2 \right)\]
\[
+ \frac{1}{(m - 1)^2} \sum_{i \neq j, i \neq k} \langle Q_{ij}^{a\beta} \rangle^2,
\]
\[
A_2 = \frac{1}{4} \left( \langle Q_{ij}^{a\beta} \rangle^2 + \frac{m - 2}{2(m - 1)} \sum_{i \neq j} \langle Q_{ij}^{a\beta} \rangle^2 + \langle Q_{ij}^{a\beta} \rangle^2 \right)
\]
\[
- \frac{1}{m - 1} \sum_{i \neq j, i \neq k} \langle Q_{ij}^{a\beta} \rangle^2.
\]

The dependence of the generating functional of the connected Green functions \(W[i]\), with \(A_1, A_2,\) and \(A_3\) insertions
\[
e^{W[i]} = \int dQ_{ij}^{a\beta} \exp \left[ - \int L_0 Q_{ij}^{a\beta} + Z_{ij} t_i A^i \right.
\]
\[
+ \left. j^{a\beta} \right] Z_{ij}^{-1/2} Q_{ij}^{a\beta}
\]

on some cutoff \(\Lambda\) is absorbed by expressing the matrix \(Z_{ij}, Z_{ij}^{\phi}\), and the bare coupling constant \(\omega_B\) in terms of the renormalized coupling constant \(\omega\) at some finite scale \(\kappa\),
\[
\omega_B = \kappa^{(-2/3)} \left[ \frac{\omega, \kappa, \Lambda}{\kappa, \Lambda} \right],
\]
\[
Z_{ij} = \left[ \frac{\omega, \kappa, \Lambda}{\kappa, \Lambda} \right],
\]
\[
Z_{ij} = \left[ \frac{\omega, \kappa, \Lambda}{\kappa, \Lambda} \right].
\]

The functions (23) and (24) are properties of the symmetric theory and were calculated before.\textsuperscript{15,21} For the
IV. EVALUATION OF THE DM Crossover Exponent

To evaluate the renormalized vertex functions with $A_i$ insertions we use dimensional regularization and minimal subtraction.\textsuperscript{16} We calculate
\[
\Gamma^{(k)}_B[1] = \Gamma^{(k)}_{B_{ij,j}}, \quad \Gamma^{(k)}_B[2] = \Gamma^{(k)}_{B_{ij,j}} + \Gamma^{(k)}_{B_{ij,j}} + \Gamma^{(k)}_{B_{ij,i}} + \Gamma^{(k)}_{B_{ij,j}},
\]
to two-loop order the superscript $(k)$ indicates that the two-point function has one insertion of the operator $A_i$. The matrix $Z_{ij} = Z_{jil}Z_{ij}$ is found requiring $Z_{ij}Z_{ij}^{(0)}$ to be finite as the regularization is removed, and $Z_{ij} = \delta_{ij}$ at the tree level. Then the minimal subtraction scheme this determines $Z_{ij}$ uniquely. From $Z_{ij}$ we construct
\[
\gamma_{ij} = \beta(\omega) Z_{ik} \left( \frac{\partial}{\partial \omega} \right) Z_{kj}^{-1}.
\]

The matrix $\gamma_{ij}$ (Eq. 30) whose eigenvalues give the crossover exponents is simply related to $\gamma_{ij}$,
\[
\gamma_{ij} = \gamma_{ij} + \gamma_{ij}.
\]

$\gamma_{ij}(\omega)$ was calculated in the symmetric theory.\textsuperscript{21} We summarize the results of the symmetric theory below,
\[
\gamma_{ij}(\omega) = -24m\omega^2 + (672m^2 - 576\omega \omega),
\]
\[
\omega_B = \kappa^2 / \left[ \omega - (18 / \epsilon) \omega \left[ m (n - 2) + 2(1 - m) \right] \right],
\]
\[
\beta(\omega) = -\omega / 2\omega + \omega - 36 + 72m\omega^3.
\]

At the symmetric fixed point,
\[
\omega^* = - \frac{\epsilon}{72(1 - 2m)} + \frac{\epsilon^2}{(1 - 2m)^3} - \frac{89m^2 + 420m - 63}{(72)^2},
\]
\[
\gamma_{ij}(\omega^*) = \frac{m\epsilon}{3(1 - 2m)} + \frac{\epsilon^2}{216(1 - 2m)^3} \left[ 33m^3 - 344m^2 + 39m \right].
\]

We evaluate the graphs in Fig. 1,
\[
\Gamma_B^{(1)}[1] = 1 + 72(n - 2)\omega_B m B_1 + (n - 2)^2 m^2 \omega_B^2 2592(3B_3 + 2B_2)
\]
\[
+ \omega_B^4 (1296m(n - 2)(n - 3)m + 1)[4B_2 + B_4],
\]
\[
\Gamma_B^{(2)}[1] = \Gamma_B^{(1)}[1] = 72B_1(n - 2)\omega_B^2
\]
\[
+ 2592(3B_3 + 2B_2)m(n - 2)^2 \omega_B^4
\]
\[
+ 1296\omega_B^4 \left[ (B_4 + 4B_2)m(n - 3)(n - 2) + 4B_2(n - 2) \right],
\]
\[
\Gamma_B^{(1)}[2] = \Gamma_B^{(2)}[3] = (n - 2)m B_4 (648) \omega_B^2,
\]
\[
\Gamma_B^{(2)}[2] = \Gamma_B^{(3)}[3] = 1,
\]
\[
\Gamma_B^{(3)}[1] = \Gamma_B^{(3)}[1] = 0.
\]

The momentum-independent part of the graphs in Fig. 1 are given by
FIG. 1. Two-point function with one-mass insertion (wavy line) graphs the correspondence between the integrals $B_j$ and the graph is the following: (a) corresponds to $B_1$; (b) and (c) to $B_2$; (b) and (d) to $B_3$, and (e) to $B_4$.

\begin{align*}
B_1 &= \frac{1}{\epsilon} - \frac{3}{4}, \\
B_2 &= \frac{1}{2\epsilon^2} - \frac{5}{\epsilon^8}, \\
B_3 &= -\frac{1}{6\epsilon^2} + \frac{11}{72\epsilon}, \\
B_4 &= \frac{1}{2\epsilon}.
\end{align*}

Substitution of (37) and (46) in (41)–(45) gives

\begin{align*}
\Gamma_B^{(1)}[1] &= 1 + \frac{a_{11}}{\epsilon} \omega^2 + a_{10} + \left[ \frac{a_{22}}{\epsilon} + \frac{a_{21}}{\epsilon} \right] \omega^4, \\
\Gamma_B^{(2)}[1] &= 1 + \frac{b_{11}}{\epsilon} \omega^2 + b_{10} + \left[ \frac{b_{22}}{\epsilon} + \frac{b_{21}}{\epsilon} \right] \omega^4, \\
\Gamma_B^{(3)}[2] &= c_{23} \frac{\omega^4}{\epsilon},
\end{align*}

with

\begin{align*}
a_{11} &= 72m(n-2), \\
a_{10} &= -54m(n-2), \\
a_{21} &= -108m(n-2)(25mn - 38m-12), \\
a_{22} &= 1296m(n-2)(mn-2), \\
b_{11} &= 72(n-2), \\
b_{10} &= -54(n-2), \\
b_{21} &= -108(n-2)(25mn - 38m-6), \\
b_{22} &= 1296(n-2)(mn-2),
\end{align*}

and

\begin{align*}
c_{23} &= 648m(n-2). \\
\bar{Z} \text{ is found by requiring finiteness of } \bar{Z}_0 \Gamma_B^{(k)}[k]. \text{ From Eq. (34) we find for } n=0,
\end{align*}

\begin{align*}
\bar{\varphi}_1 &= 0 \\
\bar{\varphi}_2 &= 0 \\
\bar{\varphi}_3 &= 0 \quad \text{also,}
\end{align*}

\begin{align*}
\bar{\varphi}_{11} &= 144m\omega^2 + \omega^4(-14688m^2 + 5184m), \\
\bar{\varphi}_{12} &= 144\omega^2 + \omega^4(-14688m + 2592), \\
\bar{\varphi}_{23} &= 2592m\omega^4.
\end{align*}

Notice that to order one loop $\Gamma_B^{(k)}[2]$ and $\Gamma_B^{(k)}[3]$ are nonsingular, therefore, $\chi_2 = 0$ and $\phi_3 = \gamma$ to that order. This equality is reminiscent of the identity between the susceptibility and the random field crossover exponent in a random ferromagnet.\footnote{Note that in Fig. 1 and the relation $\phi_3 = \gamma(\text{DM})$ breaks down to second order in $\epsilon$.}

The eigenvalues of $\chi_0(\epsilon^*)$ are [Eqs. (35) and (51)–(53)]

\begin{align*}
\lambda_1 &= 5 \frac{m\epsilon}{3(2m-1)} - \frac{\epsilon^2 m(723 m^2 + 1132m - 123)}{216(2m-1)^3}, \\
\lambda_2 &= -\frac{5em}{3(2m-1)} - \frac{\epsilon^2 m(33m^2 - 128m - 69)}{216(2m-1)^3}, \\
\lambda_3 &= \frac{5em}{3(2m-1)} - \frac{\epsilon^2 m(33m^2 - 560m + 147)}{216(2m-1)^3},
\end{align*}

and the critical exponents [Eqs. (32) and (33)]

\begin{align*}
\nu &= \frac{1}{2} + \frac{5em}{12(2m-1)} - \frac{\epsilon^2 m(123m^2 + 1432m - 123)}{864(2m-1)^3}, \\
\phi_2^{(\text{DM})} &= 1 + \frac{m\epsilon}{2m-1} + \frac{\epsilon^2 m(5m^2 - 270m + 9)}{72(2m-1)^3}.
\end{align*}

V. THE CROSSOVER EXponent:
SINGLE-ION ANISOTROPY CASE

We follow the same strategy as in the preceding section. We calculate to one-loop order the two-point vertex with single-ion anisotropy insertions. $\Gamma_B^{(1)}[1] = \Gamma_B^{(i)}[1,i]$, $\Gamma_B^{(2)}[1] = \Gamma_B^{(i)}[1,2]$, $\Gamma_B^{(3)}[2] = \Gamma_B^{(i)}[2,3]$, and the superscript $(i)$ indicates an insertion of operation $A^i$ from Eqs. (19)–(21),

\begin{align*}
\Gamma_B^{(i)}[1] &= 1 + 2m\omega^2 + 36(n-2)B_1, \\
\Gamma_B^{(i)}[2] &= \frac{m-2}{2(m-1)} [1 + m\omega^2 36(n-2)B_1], \\
\Gamma_B^{(i)}[3] &= \frac{1}{(m-1)^2}.
\end{align*}
The matrix $\mathbf{Z}$ is diagonal and given by

$$
\mathbf{Z}_{11} = 1 - 2m \frac{\omega^2 36(n-2)}{\epsilon},
$$

$$
\mathbf{Z}_{22} = 1 - m \frac{\omega^2 36(n-2)}{\epsilon},
$$

$$
\mathbf{Z}_{33} = 1.
$$

Using Eqs. (34) and (38) we find

$$
\mathbf{v}_{11} = -72(n-2)\omega^2 \epsilon^2 m ,
$$

$$
\mathbf{v}_{22} = -36(n-2)\omega^2 \epsilon^2 m ,
$$

$$
\mathbf{v}_{33} = 0.
$$

Equations (35) and (40) then give

$$
\lambda_1 = \frac{5}{3} \frac{em}{2m-1},
$$

$$
\lambda_2 = \frac{2}{3} \frac{em}{2m-1},
$$

$$
\lambda_3 = -\frac{m \epsilon}{3(2m-1)}.
$$

The first eigenvalue gives once more $1/\nu = 2 - \lambda_1$ while
from $\lambda_2, \lambda_3$ we can calculate the crossover exponent for nonrandom uniaxial single-ion anisotropy, $\phi^{(SI)}_2 = (2 - \lambda_2)\nu, \phi^{(SI)}_3 = (2 - \lambda_3)\nu$.

A detailed examination of all the diagrams which contribute to $\Gamma^{[3]}_{\beta}$ reveals that $\Gamma^{[3]}_{\beta} = +1/(m-1)^2$ to all orders in the coupling constant $\omega$. Therefore, unlike the DM case $\mathbf{v}_{33} = 0, \lambda_3 = \eta$ and the result $\phi^{(SI)}_3 = \gamma$ holds to all orders in $\epsilon$. Since $\phi^{(SI)}_2$ is smaller than $\phi^{(SI)}_3$ close to six dimensions we conclude that $\phi^{(SI)}_3$, the largest crossover exponent induced by the SI anisotropy is equal to $\gamma$, the nonlinear susceptibility exponent of the symmetric theory. This equality will break down if $\lambda_2$ becomes smaller than $\lambda_3$ at some dimension $d < 6$.

$$
\phi^{(SI)}_2 = 1 + \frac{em}{2(2m-1)}, \quad \phi^{(SI)}_3 = 1 + \frac{m \epsilon}{2m - 1}.
$$

It is interesting to point out that Eq. (71) is identical to the one-loop results of Pfeuty and Aharony for Random Exchange anisotropy even though the subspace of perturbations considered in this section is much more restricted than the one in Ref. 6.

Applying the reasoning of the preceding paragraph to the random exchange model of Pfeuty and Aharony, it is easy to see that their result, $\phi_2 = \gamma$ to lowest order in $\epsilon$, is in fact more general and holds to all orders in $\epsilon$.

VI. CONCLUSION

In this paper we studied the effect of anisotropy on the critical behavior of a $O(m)$ vector spin glass in six $\epsilon$ dimensions. We considered both single-ion anisotropies and Dyalozyinsky-Moriya interactions, which are present in metallic spin glasses like Cu$_{1-x}$Mn$_x$.

In the replica formalism they induce very different symmetry-breaking patterns of the $O(m) \times \cdots \times O(m)$ symmetry characterizing the Heisenberg fixed point. Critical fluctuations make both anisotropies more relevant than the temperature variable and have crossover exponents greater than one. Since the conduction electron mediated spin-spin interaction has always anisotropic components, the pure Heisenberg critical behavior will be hard to observe.

The SI are less relevant than the DM interactions close to the six dimensions. Since the bare DM term is also much larger than the bare SI term, the latter is likely to be unimportant for the analysis of the experiments in metallic spin glasses. In fact, the DM terms do not renormalize to lowest order and a two-loop calculation is necessary to unravel the structure of the renormalization of the theory. The SI anisotropy renormalizes to lowest order in $\epsilon$ and the largest crossover exponent is equal to $\gamma$.

The relevance of these results to real experiments is unclear. The existence of a finite temperature phase transition for a vector spin glass is still an open question and it has been suggested that the long-range nature of the RKKY interactions can be very relevant in three dimensions. If the experimentally observed critical behavior is controlled by a finite temperature fixed point it is interesting to compare the trends in the critical exponents as one moves away from dimension 6 with their experimental values.

The existence of a phase transition in finite temperature field is now a subject of active research. If there is indeed a transition below dimension 6 the shape of the Galay-Toulouse line is modified to

$$
\frac{T_g - T_{g}^{(h)}}{T_g^{(0)}} = \frac{\hbar}{h}^{2/\phi_3},
$$

where

$$
\phi_1 = 1 - \frac{e}{8} \frac{(5m+1)}{2m-1},
$$

while the de Almeida–Thouless curve becomes

$$
\frac{T_g^{(h)} - T_g^{(0)}}{T_g^{(0)}} = \frac{\hbar}{h}^{2/\phi_3}, \quad \phi_1 = 1 - \frac{3e}{4}.
$$

The crossover scale between these two behaviors occurs \[ \text{Eq. (13)} \] when $\hbar^{2\phi_3 / \phi_1} = d^2$. For the Dyalozinsky-Moriya interactions we found

$$
\frac{\phi_{2}^{(DM)}}{\phi_1} = 1 + \frac{em}{2m - 1} + \frac{3m(5m^2 - 27m + 9)}{72(2m - 1)^3},
$$

$$
2\phi_{2}^{(DM)} / \phi_1 = 1 + \frac{e(3m - 1)}{8(2m - 1)} + \frac{e^2}{576(2m - 1)^3}(123m^3 - 185m^2 - 45m - 9),
$$

while for the single ion uniaxial anisotropy we have

$$
\phi_{3}^{(SI)} / \phi_1 = 1 + \frac{me}{2m - 1},
$$

$$
2\phi_{3}^{(SI)} / \phi_1 = 1 + \frac{e(3m - 1)}{8(2m - 1)}.
Yeshurun and Sompolinsky estimated $\phi_2^{(DM)}$ to be fairly large ($\phi_2^{(DM)} \geq 5.8$). This agrees qualitatively with our results that critical fluctuations enhance $\phi_2^{(DM)}$ above its mean-field value. It would be interesting to perform more precise measurements of $\phi_2^{(DM)}$ and compare it with $\gamma$ to check whether the inequality $\phi_2^{(DM)} - \gamma > 0$ which is valid close to six dimensions is obeyed experimentally. It would also be very interesting to find an experimental realization of an isotropic vector spin glass system. Applying pressure to induce uniaxial anisotropy to this system one could check the prediction $\phi_2^{(S)} = \gamma$.

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